Adjusting surface roughness in growth processes by time delayed feedback control

M. Block and E. Schöll

Institut für Theoretische Physik, Technische Universität Berlin, D-10623 Berlin, Germany

Abstract. We propose a novel feedback control scheme in order to adjust the surface roughness during epitaxial growth processes. By means of stochastic differential equations we investigate control of the early phase of growth in situ. It is shown that the effective growth exponents $\beta$, and thus the temporal evolution of the rms (root mean square) surface roughness, can be adjusted by time delayed feedback control.

Keywords: growth phenomena; time delayed feedback control; stochastic differential equations

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INTRODUCTION

The control of surface roughness is a major issue in the field of complex growth of nanostructures. A wide range of accessible experimental setups can be characterized by the temporal evolution of the surface roughness. One of the most common methods to theoretically model growth phenomena is to use stochastic growth equations which describe the temporal evolution of a roughening surface profile [1]-[6]. The object of this paper is to propose a novel feedback control scheme in order to adjust the surface roughness during the growth process. Growth equations describe the development of a single-valued variable, $h(x,t)$, denoting the height of the surface as a function of time $t$ and position $x$. The simplest such equation is the Kardar-Parisi-Zhang (KPZ) equation [5] which describes the growth of a surface in the absence of any conservation laws:

$$\partial_t h(x,t) = v \nabla^2 h(x,t) + \frac{\lambda}{2} (\nabla h)^2 + \eta(x,t)$$

Here, $v > 0$ denotes an interface smoothing term, the nonlinear coupling $\lambda$ reflects the strength of lateral growth, and the stochastic force $\eta(x,t)$ models fluctuations in the flux of particles to the surface. We consider Gaussian white noise of intensity $D$, i.e.,

$$<\eta(x,t)> = 0$$
$$<\eta(x,t)\eta(x',t')> = 2D \delta(x-x')\delta(t-t')$$

We restrict ourselves to one spatial dimension. The main quantity is the (root mean square) surface roughness $w$, defined by

$$w^2(L,t) = \frac{1}{L} \left< \sum_x |h(x,t) - \bar{h}(t)|^2 \right>$$

where $\bar{h}$ ist the average height. The scaling exponents describing the roughness evolution of a growing surface are the growth exponent $\beta$, where $w(L,t) \sim t^\beta$ for $t \ll t_1$, the roughness exponent $\alpha$, where $w(L,t) \sim L^\alpha$ for $t \gg t_1$, and the dynamic exponent $z$, where $t_1 \sim L^z$.

METHOD

We now introduce our control method. There is experimental evidence that the lateral growth, which is governed by $\lambda$, is tunable via temperature, e.g., for chemical vapor deposition of silicon films [7]. In order to stabilize a desired effective growth exponent $\beta_0$, we propose two different control schemes which adjust $\lambda$ by a feedback loop, as shown schematically in Fig. 1. For this purpose we choose a desired value $\beta_0$ and a time delay $\tau$, and define a local exponent $\beta_{local}$ at time $t$ by

$$\beta_{local}(t) = \frac{\log w(t) - \log w(t-\tau)}{\log \tau}.$$ 

We substitute $\lambda$ in eq. (1) by $\lambda + F(t)$, where for digital control, the control force is given by

$$F(t) = \begin{cases} a, & \text{if } \beta_{local} \leq \beta_0 \\ -a, & \text{if } \beta_{local} > \beta_0 \end{cases}$$
The parameter $a$ defines the digital control step. Alternatively, for differential control,

$$F(t) \equiv K(\beta_0 - \beta_{local})$$

where $K$ is the control strength. Switching on control at time $t_0$, the parameter $\lambda$ is updated at times $t_n = t_0 + n\tau$, $n = 0, 1, 2,\ldots$, starting from an initial value $\lambda_0$, according to

$$\lambda(t) = \begin{cases} 
\lambda_0, & \text{if } t < t_0 \\
\lambda(t-\tau) + F(t), & \text{if } t = t_n \\
\lambda(t_n), & \text{if } t_n < t < t_{n+1}
\end{cases}$$

**RESULTS**

We have checked that the expected values $\alpha = 0.5$ and $\beta = 1/3$ for strong coupling and $\beta = 1/4$ for $\lambda = 0$ (Edwards-Wilkinson equation, EW) without control. The values for $\alpha$ remain unchanged if we now include control. By varying the initial parameter $\lambda_0$ and the target effective strength $\beta_0$, we can identify three different regimes (Figs. 2 and 3).

If we choose a weak initial nonlinearity $\lambda_0$, $\beta_{local}(t)$ is smaller than $\beta_0$ at early times. After the control is turned on, the control force $F(t)$ increases monotonically, and $\beta_{local}(t)$ approaches $\beta_0$ from below. If we initialize $\beta_0 = 0.25$ with large initial nonlinearity $\lambda_0 = 0.25$, the data show clearly that the control force diminishes $\lambda(t)$ so that $\beta_{local}(t)$ decreases. In the second method, the strength of the control force is determined by the parameter $K$. In Fig. 3 we present the results, using the same sets of $\beta_0$ and $\lambda_0$ as for digital control. The main conclusions are the same here: again, it is possible to stabilize effective exponents over a significant time interval.

**FIGURE 2.** Evolution of the roughness $w(t)$ and the control function $\lambda(t)$ for digital control with three initial setups for (i) $\lambda_0 = 0.25$ and $\beta_0 = 0.33$ (shifted vertically by a factor of 4), (ii) $\lambda_0 = 0.25$ and $\beta_0 = 0.25$ (shifted vertically by a factor of 2), and (iii) $\lambda_0 = 0.10$ and $\beta_0 = 0.33$. The broken lines are guides to the eye. The inset shows the control functions $\lambda(t)$ (all setups with $v = 0.10, D = 0.5, \tau = 1.0, a = 0.005$).

**FIGURE 3.** Same as Fig. 2 for differential control, with $K = 0.001$ for $\lambda_0 = 0.25$, and $K = 0.002$ for $\lambda_0 = 0.10$.

To summarize, both digital and differential control are rather successful in stabilizing desired effective growth exponents in the KPZ growth equation. For the relatively small system sizes used here, these exponents can be tuned in the range $0.25 \leq \beta \leq 0.33$, i.e., within the limits set by the EW and the KPZ equation, respectively.

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**REFERENCES**